American University of Beirut

MECH 340-Engineering Materials

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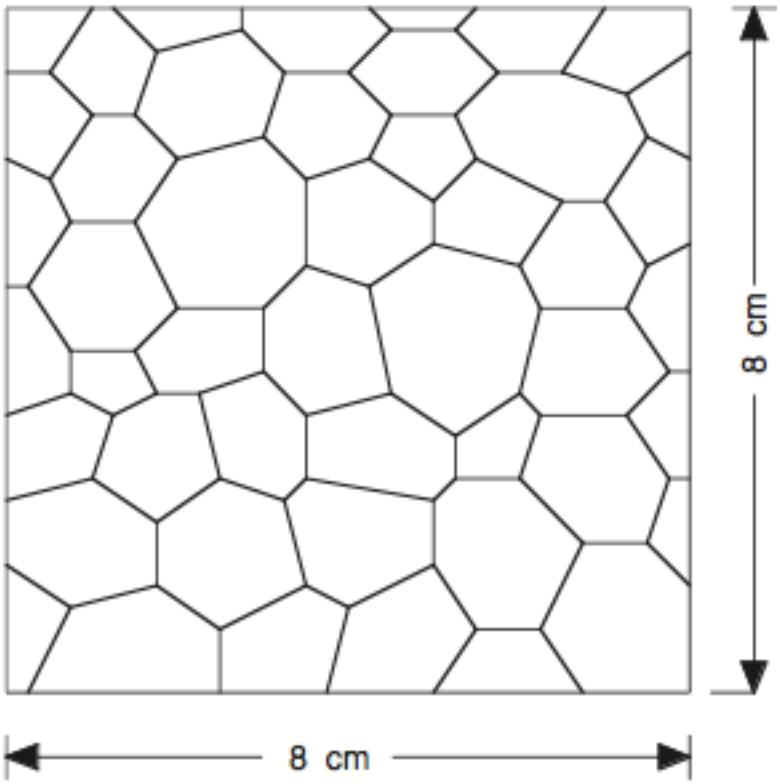
Name: **ID:**

Problem 1. Structures-15 pts

a. Figure 1 is a micrograph of an alloy at a magnification of 200×. **Find the ASTM grain size**. Count grains on the edge of the field as 1/2 and those on the corners as 1/4 in the field.



b. Draw two members of the (332) family in an orthorhombic unit cell. Draw and specify Show and specify the lattice direction of the intersection between (332) and (100).

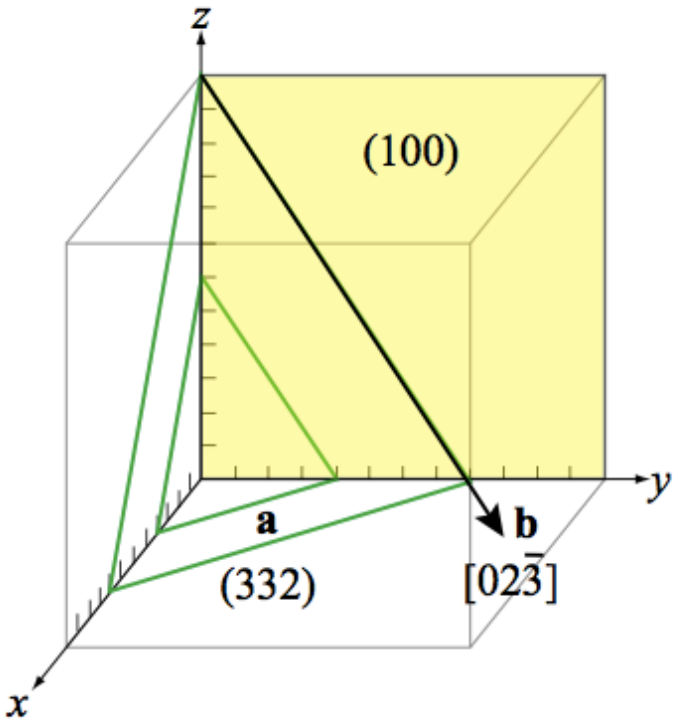
Solution:

a- There are 26 grains entirely within the field of view, 19 on the edge and 4 at the corners. Counting grains on the edge of the field as 1/2 and those on the corners as 1/4 in the field, there are 26 + 19/2 + 4/4 = 36.5 grains.

Area=64/$(2.54)^2$=9.92 in^2 then you plug into log equation to find grain size between 4 and 5 (both are accepted)

b-

a. Draw two members of the (332) family in the following unit cell.



The (332) family is derived from fractional intercepts of ⅓ along the *x*-axis, ⅓ along the *y*-axis, and ½ along the *z*-axis, as **shown and labeled** on the drawing.

b. Show and specify the lattice direction of the intersection between (332) and (100).

The line of intersection between (332) and (100) is the [02$\bar{3}$] direction (or its negative), which is shown on the illustration to fall within the *y*-*z* plane. It can be determined by noting the **coordinates of a point on the line** such as 0, ⅔, –1, and clearing fractions, or by taking the **vector cross product of the normals to the planes** [332] × [100], cross-checking with confirmation that the dot products between [02$\bar{3}$] and both plane normals [332] and [100] are zero.

Problem 2. Hall-Petch Equation and surface defects-20 pts

a. The yield strength of mild steel with an average grain size of 0.05 mm is 20,000 psi. The yield stress of the same steel with a grain size of 0.007 mm is 40,000 psi. What will be the average grain size of the same steel with a yield stress of 30,000 psi? Assume the Hall-Petch equation is valid and that changes in the observed yield stress are due to changes in grain size. (Note: 1000 psi = 6.895 MPa).

b. We would like to produce a bracket to hold ceramic bricks in place in a heat-treating furnace. The bracket should be strong, should possess some ductility so that it bends rather than fractures if overloaded, and should maintain most of its strength up to 600°C. Design the material for this bracket, considering the various crystal imperfections as the strengthening mechanism.

a-

$$\sigma_y = \sigma_0 + Kd^{-1/2}$$

Thus, for a grain size of 0.05 mm, the yield stress is

$$20{,}000 \text{ psi } (6.895 \text{ MPa})/(1000 \text{ psi}) = 137.9 \text{ MPa}$$

(Note: 1000 psi = 6.895 MPa). Using the Hall-Petch equation

$$137.9 = \sigma_0 + \frac{K}{\sqrt{0.05}}$$

For the grain size of 0.007 mm, the yield stress is 40,000 psi (6.895 MPa) (1000 psi) = 275.8 MPa. Therefore, again using the Hall-Petch equation:

$$275.8 = \sigma_0 + \frac{K}{\sqrt{0.007}}$$

Solving these two equations, $K = 18.44 \text{ MPa} \cdot \text{mm}^{1/2}$, and $\sigma_0 = 55.5$ MPa. Now we have the Hall-Petch equation as

$$\sigma_y = 55.5 + 18.44\, d^{-1/2}$$

If we want a yield stress of 30,000 psi or 206.9 MPa, the grain size should be 0.0148 mm.

b-In order to serve up to 600°C, the bracket should *not* be produced from a polymer. Instead, a metal or ceramic should be considered.

In order to have some ductility, dislocations must move and cause slip. Because slip in ceramics is difficult, the bracket should be produced from a metallic material. The metal should have a melting point well above 600°C; aluminum, with a melting point of 660°C, would not be suitable; iron, however, would be a reasonable choice.

We can introduce point, line, and surface imperfections into the iron to help produce strength, but we wish the imperfections to be stable as the service temper- ature increases. As shown in Chapter 5, grains can grow at elevated temperatures, reducing the number of grain boundaries and causing a decrease in strength. As indicated in Chapter 8, dislocations may be annihilated at elevated temperatures— again, reducing strength. The number of vacancies depends on temperature, so con- trolling these crystal defects may not produce stable properties.

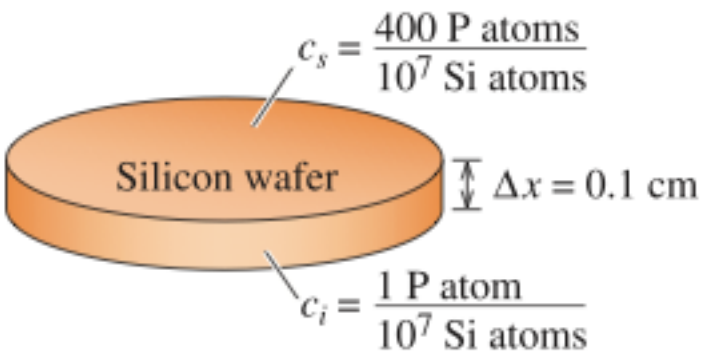
The number of interstitial or substitutional atoms in the crystal does not, however, change with temperature. We might add carbon to the iron as interstitial atoms or substitute vanadium atoms for iron atoms at normal lattice points. These point defects continue to interfere with dislocation movement and help to keep the strength stable.

Problem 3. Diffusion-15 pts

One step in manufacturing transistors, which function as electronic switches in integrated circuits, involves diffusing impurity atoms into a semiconductor material such as silicon (Si). Suppose a silicon wafer 0.1 cm thick, which originally contains one phosphorus atom for every 10 million Si atoms, is treated so that there are 400 phosphorous (P) atoms for every 10 million Si atoms at the surface

Calculate the concentration gradient (a) in atomic percent/cm and (b) in $\frac{atoms}{cm^3.cm}$.

The lattice parameter of silicon is 5.4307Å and the structure of diamond has 8 atoms/cell. ($D=D_0\exp(-Q/RT)$, Q in cal/mol and R=1.987 cal/mol.k)



a. First, let's calculate the initial and surface compositions in atomic percent:

$$c_i = \frac{1\text{ P atom}}{10^7\text{ atoms}} \times 100 = 0.00001\text{ at\% P}$$

$$c_s = \frac{400\text{ P atoms}}{10^7\text{ atoms}} \times 100 = 0.004\text{ at\% P}$$

$$\frac{\Delta c}{\Delta x} = \frac{0.00001 - 0.004\text{ at\% P}}{0.1\text{ cm}} = -0.0399\frac{\text{at\% P}}{\text{cm}}$$

b. To find the gradient in terms of $\frac{\text{atoms}}{\text{cm}^3\cdot\text{cm}}$, we must find the volume of the unit cell. The crystal structure of Si is diamond cubic (DC). The lattice parameter is 5.4307×10^{-8} cm. Thus,

$$V_{\text{cell}} = (5.4307 \times 10^{-8}\text{ cm})^3 = 1.6 \times 10^{-22}\frac{\text{cm}^3}{\text{cell}}$$

The volume occupied by 10^7 Si atoms, which are arranged in a DC structure with 8 atoms/cell, is

$$V = \left[\frac{10^7\text{ atoms}}{8\frac{\text{atoms}}{\text{cell}}}\right]\left[1.6 \times 10^{-22}\left(\frac{\text{cm}^3}{\text{cell}}\right)\right] = 2 \times 10^{-16}\text{ cm}^3$$

The compositions in atoms/cm^3 are

$$c_i = \frac{1 \text{ P atom}}{2 \times 10^{-16} \text{ cm}^3} = 0.005 \times 10^{18} \text{ P}\left(\frac{\text{atoms}}{\text{cm}^3}\right)$$

$$c_s = \frac{400 \text{ P atoms}}{2 \times 10^{-16} \text{ cm}^3} = 2 \times 10^{18} \text{ P}\left(\frac{\text{atoms}}{\text{cm}^3}\right)$$

Thus, the composition gradient is

$$\frac{\Delta c}{\Delta x} = \frac{0.005 \times 10^{18} - 2 \times 10^{18} \text{ P}\left(\frac{\text{atoms}}{\text{cm}^3}\right)}{0.1 \text{ cm}}$$
$$= -1.995 \times 10^{19} \text{ P}\left(\frac{\text{atoms}}{\text{cm}^3 \cdot \text{cm}}\right)$$

Problem 4. Diffusion-15 pts

Types of Diffusion In **volume diffusion,** the atoms move through the crystal from one regular or interstitial site to another. Because of the surrounding atoms, the activation energy is large and the rate of diffusion is relatively slow.

Atoms can also diffuse along boundaries, interfaces, and surfaces in the material. Atoms diffuse easily by **grain boundary diffusion,** because the atom packing is disordered and less dense in the grain boundaries. Because atoms can more easily squeeze their way through the grain boundary, the activation energy is low (Table 5-2). **Surface diffusion** is easier still because there is even less constraint on the diffusing atoms at the surface.

TABLE 5-2 ■ *The effect of the type of diffusion for thorium in tungsten and for self-diffusion in silver*

Diffusion Type	Diffusion Coefficient (D)			
	Thorium in Tungsten		Silver in Silver	
	D_0 (cm^2/s)	Q (cal/mol)	D_0 (cm^2/s)	Q (cal/mol)
Surface	0.47	66,400	0.068	8,900
Grain boundary	0.74	90,000	0.24	22,750
Volume	1.00	120,000	0.99	45,700

Consider a diffusion couple between pure tungsten and a tungsten alloy containing 1 at% thorium. After several minutes of exposure at 2000°C, a transition zone of 0.01 cm thickness is established. What is the flux of thorium atoms at this time if diffusion is due to (a) volume diffusion, (b) grain boundary diffusion, and (c) surface diffusion? (See Table 5-2.)

SOLUTION

The lattice parameter of BCC tungsten is 3.165 Å. Thus, the number of tungsten atoms/cm^3 is

$$\frac{\text{W atoms}}{\text{cm}^3} = \frac{2 \text{ atoms/cell}}{(3.165 \times 10^{-8})^3 \text{ cm}^3/\text{ cell}} = 6.3 \times 10^{22}$$

In the tungsten-1 at% thorium alloy, the number of thorium atoms is

$$c_{\text{Th}} = (0.01)(6.3 \times 10^{22}) = 6.3 \times 10^{20} \text{ Th } \frac{\text{atoms}}{\text{cm}^3}$$

In the pure tungsten, the number of thorium atoms is zero. Thus, the concentration gradient is

$$\frac{\Delta c}{\Delta x} = \frac{0 - 6.3 \times 10^{20} \frac{\text{atoms}}{\text{cm}^2}}{0.01 \text{ cm}} = -6.3 \times 10^{22} \text{ Th } \frac{\text{atoms}}{\text{cm}^3 \cdot \text{cm}}$$

a. Volume diffusion

$$D = 1.0 \frac{\text{cm}^2}{\text{s}} \exp\left(\frac{-120{,}000 \frac{\text{cal}}{\text{mol}}}{\left(1.987 \frac{\text{cal}}{\text{mol} \cdot \text{K}}\right)(2273 \text{ K})}\right) = 2.89 \times 10^{-12} \text{ cm}^2/\text{s}$$

$$J = -D\frac{\Delta c}{\Delta x} = -\left(2.89 \times 10^{-12} \frac{\text{cm}^2}{\text{s}}\right)\left(-6.3 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3 \cdot \text{cm}}\right)$$
$$= 18.2 \times 10^{10} \frac{\text{Th atoms}}{\text{cm}^2 \cdot \text{s}}$$

b. Grain boundary diffusion

$$D = 0.74 \frac{\text{cm}^2}{\text{s}} \exp\left(\frac{-90{,}000 \frac{\text{cal}}{\text{mol}}}{\left(1.987 \frac{\text{cal}}{\text{mol} \cdot \text{K}}\right)(2273 \text{ K})}\right) = 1.64 \times 10^{-9} \text{ cm}^2/\text{s}$$

$$J = -\left(1.64 \times 10^{-9} \frac{\text{cm}^2}{\text{s}}\right)\left(-6.3 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3 \cdot \text{cm}}\right) = 10.3 \times 10^{13} \frac{\text{Th atoms}}{\text{cm}^2 \cdot \text{s}}$$

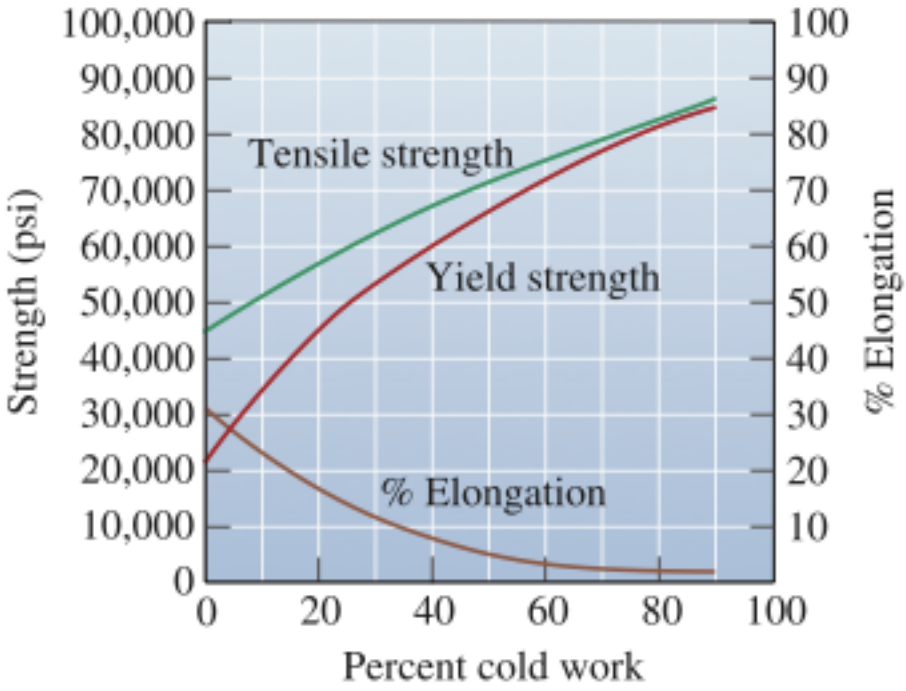
c. Surface diffusion

$$D = 0.47 \frac{\text{cm}^2}{\text{s}} \exp\left(\frac{-66{,}400 \frac{\text{cal}}{\text{mol}}}{\left(1.987 \frac{\text{cal}}{\text{mol} \cdot \text{K}}\right)(2273 \text{ K})}\right) = 1.94 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$J = -\left(1.94 \times 10^{-7} \frac{\text{cm}^2}{\text{s}}\right)\left(-6.3 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3 \cdot \text{cm}}\right) = 12.2 \times 10^{15} \frac{\text{Th atoms}}{\text{cm}^2 \cdot \text{s}}$$

Problem 5. Cold work-15 pts

Design a manufacturing process to produce a 0.1-cm-thick copper plate having at least 65,000 psi tensile strength, 60,000 psi yield strength, and 5% elongation.



Solution:

From Figure, we need at least 35% cold work to produce a tensile strength of 65,000 psi and 40% cold work to produce a yield strength of 60,000 psi, but we need less than 45% cold work to meet the 5% elongation requirement. Therefore, any cold work between 40% and 45% gives the required mechanical properties.

To produce the plate, a cold-rolling process would be appropriate. The original thickness of the copper plate prior to rolling can be calculated assuming that the width of the plate does not change. Because there is a range of allow- able cold work—between 40% and 45%—there is a range of initial plate thicknesses:

$$\%\,CW_{min} = 40 = \left[\frac{t_{min} - 0.1\text{ cm}}{t_{min}}\right] \times 100, \quad \therefore t_{min} = 0.167\text{ cm}$$

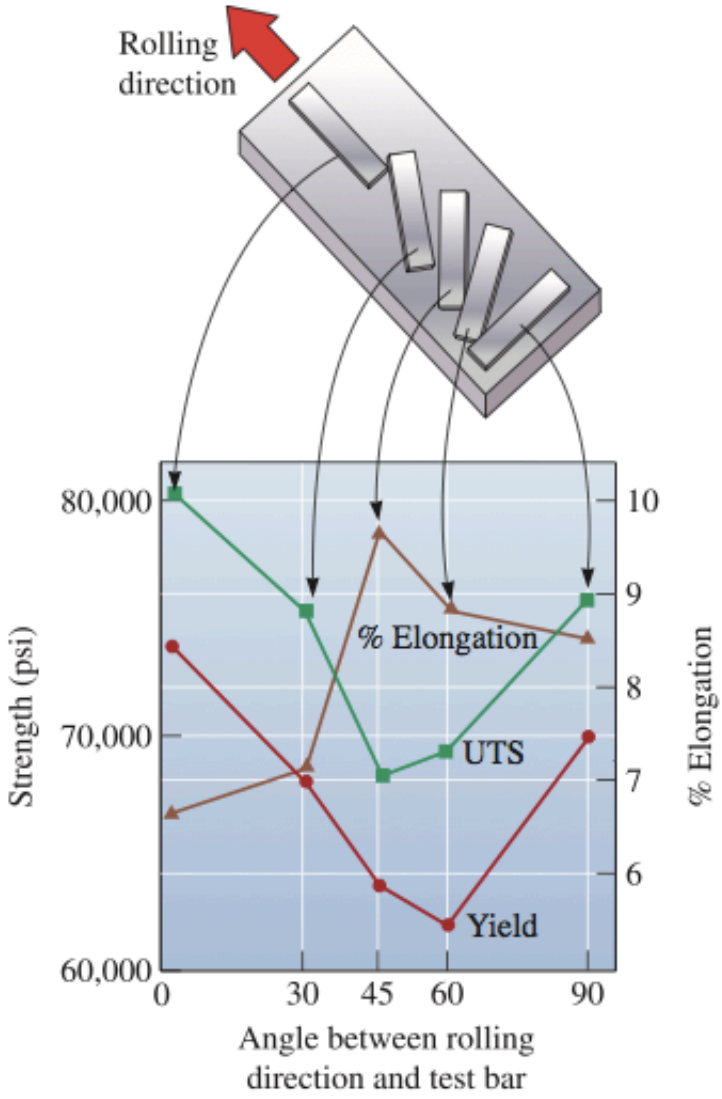
$$\%\,CW_{max} = 45 = \left[\frac{t_{max} - 0.1\text{ cm}}{t_{max}}\right] \times 100, \quad \therefore t_{max} = 0.182\text{ cm}$$

To produce the 0.1-cm copper plate, we begin with a 0.167- to 0.182-cm copper plate in the softest possible condition, then cold roll the plate 40% to 45% to achieve the 0.1 cm thickness.

Problem 6. Cold Work and Burger vector (20 pts)

a. In processes such as rolling, grains become oriented in a preferred crystallo- graphic direction and plane, giving a **sheet texture.** The properties of a rolled sheet or plate depend on the direction in which the property is measured. The Figure below summarizes the tensile properties of a cold-worked aluminim-lithium (Al-Li) alloy.

- Explain the curve
- What mechanical property is mainly highlighted under this experiment
- This method is used in aerospace applications. Why?



b. In an FCC crystal, active slip systems consist of close-packed {111} planes and the <011> directions in those planes. Following an electron microscopy

investigation, it is discovered that a dislocation in a Ni-based superalloy (FCC) has a Burgers vector

$$\vec{b} = \frac{a}{2}[\bar{1}10]$$

and a line direction vector

$$\vec{\xi} = [\bar{1}\bar{1}2]$$

Is this an edge or screw dislocation? (Hint: use dot product concept)

What is its slip plane?

Solution:

Part 1:

a-In processes such as rolling, grains become oriented in a preferred crystallo- graphic direction and plane, giving a **sheet texture.** The properties of a rolled sheet or plate depend on the direction in which the property is measured. Figure 8-10 summa- rizes the tensile properties of a cold-worked aluminim-lithium (Al-Li) alloy. For this alloy, strength is highest parallel to the rolling direction, whereas ductility is highest at a 45° angle to the rolling direction. The strengthening that occurs by the development of anisotropy or of a texture is known as **texture strengthening.** As pointed out in Chapter 6, the Young's modulus of materials also depends upon crystallographic directions in sin- gle crystals. For example, the Young's modulus of iron along [111] and [100] directions is =260 and 140 GPa, respectively. The dependence of yield strength on texture is even stronger. Development of texture not only has an effect on mechanical properties but also on magnetic and other properties of materials. For example, grain-oriented magnetic steels made from about 3% Si and 97% Fe used in transformer cores are textured via thermo-mechanical processing so as to optimize their electrical and magnetic properties.

b-mechanical property=anisotropy

c-texture strengthening with high yield is adequate to aerospace application when high strength is needed

Part 2-

Dot product is zero so edge dislocation

Slip plane:111